A damage-based condensation method to condense wave bins for tendon fatigue analysis

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ABSTRACT

Tendons are the main station keeping system to maintain Tension Leg Platforms (TLPs) in place. One of the key tendon design parameters is its fatigue life and wave is an important source for tendon fatigue damage. The wave scatter diagram represents the joint probabilities of the significant wave heights and the wave peak periods for different wave headings in one particular area. The number of sea states in the wave scatter diagram is usually too large, so that the full wave scatter diagram is usually condensed into several wave bins to reduce the computational burden. Tendon fatigue results are quite sensitive to the selection of the equivalent fatigue bins and their associated probabilities in each wave direction. This paper introduces a Damage-Based Condensation Method (DBCM) to condense the wave scatter diagram, which can significantly reduce the computation cost while still maintaining sufficient accuracy. The results of frequency domain tendon fatigue analysis show that the results from the DBCM have a good agreement with the results from the full wave scatter diagram.

Keywords: TLP tendon, Wave fatigue analysis, Wave scatter diagram, Damage-based condensation method.

1 Introduction

The Tension Leg Platform (TLP) is widely used in the oil & gas industry in the water depth ranging from about 400m to 1500m (Wu et al., 2015). TLP is recognized by its favorable responses especially in the heave, roll and pitch degrees of freedom. The tendon is a system of components, which form a link between the TLP platform hull and the subsea foundation for the purpose of mooring the TLP (API, 2010). The tendon system restrains the motion of the platform in response to winds, waves, currents, and tides to within the specified limits.

One of the key tendon design requirements is fatigue life, since the tendon has to subsist in the environment during the whole design life. Cyclic loads acting on the tendons can cause fatigue
damage. The tendon fatigue damage may come from different sources, such as the environmental loads, mechanical vibration, vortex induced vibration (VIV), etc. One of the main fatigue sources is the wave load.

A wave scatter diagram is typically developed to represent the range of environments expected during the service life along with their probability of occurrences. It represents the joint probabilities of significant wave heights and wave peak periods for different wave headings in one particular area.

There are two common methods to calculate the fatigue damage, one is time domain analysis and the other is frequency domain analysis. The time domain analysis is more accurate since non-linear effects from waves and the tendon systems can be included. Also it has the advantages of combining the fatigue damage from axial tension and from bending moment with accurate phasing. The drawback of the time domain approach is that it can be time-consuming compared to frequency domain approach. Since the total number of sea-states in the wave scatter diagram is typically excessive, it is impractical to perform the time domain analyses for all sea-states in the scatter diagram. Therefore, it is a common practice to condensate the full wave scatter diagram into several wave bins, and performs the time domain analyses only for those condensed wave bins.

Different condensation methods may result in different wave bins as well as different tendon fatigue damage. The good condensed wave scatter diagram is the one that produces similar tendon fatigue damage as the full scatter diagram with sufficient accuracy. The common method that is used to condense the full scatter diagram typically relies on producing equivalent significant wave height and period based on proportional relationships without taking into account the actual tendon stresses from each of the actual bins in the full scatter diagram. Therefore, the fatigue damage calculated from the condensed bins is typically inaccurate and sometimes contains significant errors in certain sea states.

In this paper, the proposed “damage-based condensation method” is introduced to provide a robust binning approach that leads to an equivalent fatigue damage to that of the full wave scatter diagrams with sufficient accuracy.

2 Methodology

The deterministic fatigue approach is widely used to calculate the tendon fatigue damage. It uses an S-N curve, which gives the number of cycles to failure for a specific structural detail as a function of constant stress range. For each sea-state, it is assumed that the hotspot stress amplitude follows the Rayleigh distribution. Then the accumulated tendon wave fatigue damage with a single-slope S-N curve is calculated in frequency domain by equation (1) (DNV, 2008).

\[
D = \frac{T \ast (8\sigma_{ms}^2SCF^2)^{m/2}}{KT_{av}} \Gamma \left(1 + \frac{m}{2}\right)
\]  

(1)
where,

\[ D = \text{wave fatigue damage due to an individual sea state} \]
\[ T = \text{individual sea state duration time} \]
\[ \sigma_{rms} = \text{root mean square (RMS) of the hotspot stress amplitude} \]
\[ m = \text{negative inverse slope of the S-N curve} \]
\[ T_{av} = \text{stress average zero up-crossing period} \]
\[ K = \text{interception parameter of the S-N curve} \]
\[ \Gamma(\cdot) = \text{complete Gamma function} \]
\[ SCF = \text{stress concentration factor} \]

The tendon hotspot stress spectrum is used to describe the dynamic stress response. It can be derived based on equation (2). The RMS of the tendon hotspot stress and the average up-crossing period can be obtained from the stress spectrum by equation (3) and (4), respectively.

\[
\sigma_S(\omega) = |\sigma_{RAO}(\omega)|^2 \ast W\nu_S(\omega) \tag{2}
\]
\[
\sigma_{rms} = \sqrt{\sigma_{M0}} \tag{3}
\]
\[
T_{av} = 2\pi \frac{\sigma_{M0}}{\sigma_{M2}} \tag{4}
\]

where

\[ \sigma_S(\omega) = \text{hotspot stress spectrum at frequency } \omega \]
\[ \sigma_{RAO}(\omega) = \text{hotspot stress RAO at frequency } \omega \]
\[ W\nu_S(\omega) = \text{wave spectrum at frequency } \omega \]
\[ \sigma_{M0} = \text{zeroth moment of hotspot stress spectrum} \]
\[ \sigma_{M2} = \text{second moment of hotspot stress spectrum} \]

The wave induced fatigue damages from all sea-states are calculated based on a damage summation by Miner approach as equation (2).

\[ D_t = \sum D_i \tag{5} \]

where

\[ D_i = \text{wave fatigue damage due to an individual sea state} \]
\[ D_t = \text{total wave fatigue damage} \]

Typically, the hotspot stress RAO changes with frequency. Therefore, even with the same wave height, different wave period could result in different fatigue damage. Therefore, to find a representative condensed wave bin, it is recommended to determine the condensed wave peak period first, then to find the corresponding condensed significant wave height.
Assuming that the wave spectrum follows the JONSWAP wave spectrum, the wave average zero up-crossing period $T_z$ is calculated from the wave peak period by equation (6) (DNV, 2007). The number of waves from each sea state is calculated by equation (7). The condensed wave average zero up-crossing period $\overline{T_z}$ that shall generate the same number of waves is calculated by equation (8). The condensed wave peak period $\overline{T_p}$ is then calculated by equation (9).

\begin{align*}
T_z(i) &= (0.6673 + 0.05037 \gamma - 0.00623 \gamma^2 + 0.0003341 \gamma^3) \times T_p(i) \\
N_{wv}(i) &= \frac{T_i}{T_z(i)} \\
\overline{T_z} &= \frac{\sum T_i}{\sum N_{wv}(i)} \\
\overline{T_p} &= \frac{\overline{T_z}}{(0.6673+0.05037\gamma-0.00623\gamma^2+0.0003341\gamma^3)}
\end{align*}

where

- $T_p(i)$ = wave peak period of an individual sea state
- $\gamma$ = JONSWAP peak shape factor of an individual sea state
- $T_i$ = time duration for an individual sea state
- $T_z(i)$ = wave average up-crossing period of individual sea state
- $N_{wv}(i)$ = wave number of an individual sea state
- $\overline{T_z}$ = condensed wave average up-crossing period
- $\overline{T_p}$ = condensed wave peak period

The condensed significant wave height $\overline{Hs}$ is then calculated based on the procedure below to ensure that the produced damage from this equivalent bin is close to the total damage from original sea states. This method is to find a pair of $\overline{Hs}$ and $\overline{T_p}$ for each wave heading in the bins.

1) Calculate the tendon hotspot fatigue damage based on equation (1) ~ (5).
2) Calculate the condensed wave peak period based on equation (6) ~ (9)
3) Select several condensed wave heights $\overline{Hs}$, which shall cover all possible range of the condensed wave heights.

In this study, 400 selected condensed wave heights $\overline{Hs}$, which range from 0 to $Hs_{max} + \Delta Hs/2$, are used.

where

- $Hs_{min}$ = minimum $Hs$ in the sea states.
- $Hs_{max}$ = maximum $Hs$ in the sea states.
- $\Delta Hs = Hs_{max} - Hs_{min}$
Calculate the tendon fatigue damage based on equation (1) ~ (4) for each $H_s$ with the same $T_p$. Compare this fatigue damage with the damage obtained in step 1.

4) The $H_s$, which results in the least error in fatigue damage, is selected as the condensed significant wave height.

3 Results

A case study is prepared to demonstrate the application of this proposed methodology and its accuracy. The assumed tendon hotspot stress RAO for different wave headings are presented in Fig. 1.

Table 1 presents the wave scatter diagram used in this study. This wave scatter diagram consists of 6 significant wave heights, 2 peak periods and 8 headings.

The API-X S-N curve is used in this study. The SCF effect is ignored in this study as it should not affect the conclusion. The damage-based condensation method (DBCM) has been used and the condensed $H_s$, $T_p$ and the corresponding errors in the fatigue damage for each heading are estimated and listed in Table 2. The damage error when using the DBCM is found to be less than 1%, indicating an excellent accuracy of the proposed method. The fatigue damage computational time based on the DBCM is found to be only 1/12 of that using the full wave scatter diagram which demonstrates the high efficiency of this novel approach.
Table 1 Wave scatter diagram example.

<table>
<thead>
<tr>
<th>Probability (%)</th>
<th>0deg</th>
<th>45deg</th>
<th>90deg</th>
<th>135deg</th>
<th>180deg</th>
<th>225deg</th>
<th>270deg</th>
<th>315deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tp (s)</td>
<td>6.5</td>
<td>7.5</td>
<td>6.5</td>
<td>7.5</td>
<td>6.5</td>
<td>7.5</td>
<td>6.5</td>
<td>7.5</td>
</tr>
<tr>
<td>Hs (ft)</td>
<td>8.58</td>
<td>6.33</td>
<td>5.68</td>
<td>5.08</td>
<td>7.08</td>
<td>6.90</td>
<td>7.78</td>
<td>7.40</td>
</tr>
<tr>
<td>Damage Error</td>
<td>-0.25%</td>
<td>-0.50%</td>
<td>-0.56%</td>
<td>-0.74%</td>
<td>0.43%</td>
<td>-0.15%</td>
<td>-0.18%</td>
<td>0.44%</td>
</tr>
</tbody>
</table>

The traditional method used to condense the full scatter diagram estimates $\overline{Hs}$ as equation (10). With the same $\overline{Tp}$ as above, the condensed $\overline{Hs}$ and the corresponding fatigue damage errors from the full wave scatter diagram in frequency domain when using the traditional method are listed in Table 3.

$$\overline{Hs} = \frac{\sum T_i H_{si}^m}{\sum T_i}$$ (10)

Table 2 Condensed $\overline{Hs}$, $\overline{Tp}$ and damage errors for each heading.

<table>
<thead>
<tr>
<th>Heading</th>
<th>0deg</th>
<th>45deg</th>
<th>90deg</th>
<th>135deg</th>
<th>180deg</th>
<th>225deg</th>
<th>270deg</th>
<th>315deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\overline{Hs}$ (ft)</td>
<td>6.83</td>
<td>6.83</td>
<td>6.81</td>
<td>6.79</td>
<td>6.87</td>
<td>6.85</td>
<td>6.89</td>
<td>6.93</td>
</tr>
<tr>
<td>$\overline{Tp}$ (s)</td>
<td>6.5</td>
<td>7.5</td>
<td>6.5</td>
<td>7.5</td>
<td>6.5</td>
<td>7.5</td>
<td>6.5</td>
<td>7.5</td>
</tr>
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<td>-0.25%</td>
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<td>-0.56%</td>
<td>-0.74%</td>
<td>0.43%</td>
<td>-0.15%</td>
<td>-0.18%</td>
<td>0.44%</td>
</tr>
</tbody>
</table>

Comparing error values from Table 2 and 3, it can be clearly seen that the new introduced “damage based condense method” achieves excellent accuracy while the traditional method could lead to much larger fatigue damage errors with unacceptable accuracy.

3 Conclusions

In this paper, a novel approach is introduced to condense the full wave scatter diagram into fewer number of fatigue bins while maintaining the produced fatigue damage accuracy and
significantly reducing the computational time. The novel approach is called the Damage-Based Condensation Method (DBCM). Unlike the traditional method, the DBCM utilizes the fatigue damage as the basis for condensing the wave scatter diagram. A case study is presented where the DBCM is applied and the fatigue damage is compared to that when the full wave scatter diagram is used. The fatigue damage error using DBCM is found to be less than 1% with a twelve times reduction in the computational time. Therefore, the DBCM can significantly reduce the computation cost while maintaining excellent accuracy.

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References


Biographies

Cheng Peng is a Naval Architect at the INTECSEA WorleyParsons Group. He received his BS and MS degrees in Naval Architecture and Ocean Engineering from Tianjin University in 2005 and 2007, respectively, and his PhD degree in Ocean Engineering from the Texas A&M University in 2015. His experiences include design and global performance analysis of offshore platforms and offshore wind turbines. He is a member of International Association of Ocean Engineers (IAOE).